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# Differential Equations

## TOPIC 1 Ordinary Differential Equations, Order & Degree of Differential Equations, Formation of Differential Equations

1. The differential equation of the family of curves, $x^2 = 4b(y + b), b \in R$, is: **[Jan. 8, 2020 (II)]**

   (a) $x(y')^2 = x + 2yy'$ (b) $x(y')^2 = 2yy' - x$

   (c) $xy'' = y'$ (d) $x(y')^2 = x - 2yy'$

2. The differential equation representing the family of ellipse having foci either on the x-axis or on the y-axis centre at the origin and passing through the point (0, 3) is: **[Online April 16, 2018]**

   (a) $xyy' + y^2 - 9 = 0$ (b) $x + yy'' = 0$

   (c) $xyy'' + x(y')^2 - yy' = 0$ (d) $xyy' - y^2 + 9 = 0$

3. If the differential equation representing the family of all circles touching x-axis at the origin is

   $\left(x^2 - y^2\right)\frac{dy}{dx} = g(x)y$, then g(x) equals: **[Online April 9, 2014]**

   (a) $\frac{1}{2}x$ (b) $2x^2$

   (c) $2x$ (d) $\frac{1}{2}x^2$

4. **Statement-1:** The slope of the tangent at any point P on a parabola, whose axis is the axis of $x$ and vertex is at the origin, is inversely proportional to the ordinate of the point P.

   **Statement-2:** The system of parabolas $y^2 = 4ax$ satisfies a differential equation of degree 1 and order 1. **[Online April 9, 2013]**

   (a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.

   (b) Statement-1 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for statement-1.

   (c) Statement-1 is true; Statement-2 is false.

   (d) Statement-1 is false; Statement-2 is true.

5. **Statement 1:** The degrees of the differential equations $\frac{dy}{dx} + y^2 = x$ and $\frac{d^2y}{dx^2} + y = \sin x$ are equal.

   **Statement 2:** The degree of a differential equation, when it is a polynomial equation in derivatives, is the highest positive integral power of the highest order derivative involved in the differential equation, otherwise degree is not defined. **[Online May 12, 2012]**

   (a) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

   (b) Statement 1 is false, Statement 2 is true.

   (c) Statement 1 is true, Statement 2 is false.

   (d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

6. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where $c_1$, and $c_2$ are arbitrary constants, is **[2009]**

   (a) $y'' = y'y$ (b) $yy'' = y'$

   (c) $yy'' = (y')^2$ (d) $y' = y^2$

7. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is **[2009]**

   (a) $(x-2)y'^2 = 25 - (y-2)^2$

   (b) $(y-2)y'^2 = 25 - (y-2)^2$

   (c) $(y-2)^2 y'^2 = 25 - (y-2)^2$

   (d) $(x-2)^2 y'^2 = 25 - (y-2)^2$

8. The differential equation of all circles passing through the origin and having their centres on the x-axis is **[2007]**

   (a) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (b) $y^2 = x^2 - 2xy\frac{dy}{dx}$

   (c) $x^2 = y^2 + xy\frac{dy}{dx}$ (d) $x^2 = y^2 + 3xy\frac{dy}{dx}$

**9.** The differential equation whose solution is $Ax^2 + By^2 = 1$ where A and B are arbitrary constants is of **[2006]**
(a) second order and second degree
(b) first order and second degree
(c) first order and first degree
(d) second order and first degree

**10.** The differential equation representing the family of curves $y^2 = 2c(x+\sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows : **[2005]**
(a) order 1, degree 2 (b) order 1, degree 1
(c) order 1, degree 3 (d) order 2, degree 2

**11.** The differential equation for the family of circle $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is **[2004]**
(a) $(x^2+y^2)y' = 2xy$ (b) $2(x^2+y^2)y' = xy$
(c) $(x^2-y^2)y' = 2xy$ (d) $2(x^2-y^2)y' = xy$

**12.** The degree and order of the differential equation of the family of all parabolas whose axis is $x$ - axis, are respectively. **[2003]**
(a) 2, 3 (b) 2, 1
(c) 1, 2 (d) 3, 2.

**13.** The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ are **[2002]**
(a) $(1, \frac{2}{3})$ (b) (3, 1)
(c) (3, 3) (d) (1, 2)

## TOPIC 2 General & Particular Solution of Differential Equation, Solution of Differential Equation by the Method of Separation of Variables, Solution of Homogeneous Differential Equations

**14.** The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$ is : **[Sep. 06, 2020 (I)]**
(where C is a constant of integration)
(a) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$
(b) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$
(c) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$
(d) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

**15.** If $y = \left(\frac{2}{\pi}x - 1\right)$ cosec $x$ is the solution of the differential equation, $\frac{dy}{dx} + p(x)y = \frac{2}{\pi}\text{cosec}\, x$, $0 < x < \frac{\pi}{2}$, then the function p(x) is equal to: **[Sep. 06, 2020 (II)]**
(a) cot $x$ (b) cosec $x$
(c) sec $x$ (d) tan $x$

**16.** If $y = y(x)$ is the solution of the differential equation $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx} + e^x = 0$ satisfying $y(0) = 1$, then a value of $y(\log_e 13)$ is : **[Sep. 05, 2020 (I)]**
(a) 1 (b) – 1
(c) 0 (d) 2

**17.** The solution of the differential equation $\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$ is : **[Sep. 04, 2020 (II)]**
(where $C$ is a constant of integration.)
(a) $x - \frac{1}{2}(\log_e(y+3x))^2 = C$
(b) $x - \log_e(y+3x) = C$
(c) $y + 3x - \frac{1}{2}(\log_e x)^2 = C$
(d) $x - 2\log_e(y+3x) = C$

**18.** Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function such that $f(1) = e$ and $\lim_{t\to x}\frac{t^2f^2(x) - x^2f^2(t)}{t-x} = 0$.
If $f(x) = 1$, then $x$ is equal to : **[Sep. 04, 2020 (II)]**
(a) $\frac{1}{e}$ (b) $2e$
(c) $\frac{1}{2e}$ (d) $e$

19. The solution curve of the differential equation, $(1+e^{-x})(1+y^2)\frac{dy}{dx}=y^2$, which passes through the point (0, 1), is : **[Sep. 03, 2020 (I)]**

(a) $y^2+1=y\left(\log_e\left(\frac{1+e^{-x}}{2}\right)+2\right)$

(b) $y^2+1=y\left(\log_e\left(\frac{1+e^{x}}{2}\right)+2\right)$

(c) $y^2=1+y\log_e\left(\frac{1+e^{x}}{2}\right)$

(d) $y^2=1+y\log_e\left(\frac{1+e^{-x}}{2}\right)$

20. If $x^3dy+xy\,dx=x^2dy+2y\,dx;\ y(2)=e$ and $x>1$, then $y(4)$ is equal to : **[Sep. 03, 2020 (II)]**

(a) $\frac{3}{2}+\sqrt{e}$ (b) $\frac{3}{2}\sqrt{e}$

(c) $\frac{1}{2}+\sqrt{e}$ (d) $\frac{\sqrt{e}}{2}$

21. Let $y=y(x)$ be the solution of the differential equation, $\frac{2+\sin x}{y+1}\cdot\frac{dy}{dx}=-\cos x,\ y>0,\ y(0)=1$. If $y(\pi)=a$ and $\frac{dy}{dx}$ at $x=\pi$ is $b$, then the ordered pair $(a, b)$ is equal to : **[Sep. 02, 2020 (I)]**

(a) $\left(2,\frac{3}{2}\right)$ (b) $(1,-1)$

(c) $(1,1)$ (d) $(2,1)$

22. If a curve $y=f(x)$, passing through the point (1, 2), is the solution of the differential equation, $2x^2dy=(2xy+y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to : **[Sep. 02, 2020 (II)]**

(a) $\frac{1}{1+\log_e 2}$ (b) $\frac{1}{1-\log_e 2}$

(c) $1+\log_e 2$ (d) $\frac{-1}{1+\log_e 2}$

23. If $f2(x)=\tan^{-1}(\sec x+\tan x), -\frac{\pi}{2}<x<\frac{\pi}{2}$, and $f(0)=0$, then $f(1)$ is equal to: **[Jan. 9, 2020 (I)]**

(a) $\frac{\pi+1}{4}$ (b) $\frac{1}{4}$

(c) $\frac{\pi-1}{4}$ (d) $\frac{\pi+2}{4}$

24. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2};\ y(1)=1;$ then a value of $x$ satisfying $y(x)=e$ is: **[Jan. 9, 2020 (II)]**

(a) $\frac{1}{2}\sqrt{3}\,e$ (b) $\frac{e}{\sqrt{2}}$

(c) $\sqrt{2}\,e$ (d) $\sqrt{3}\,e$

25. Let $f(x)=(\sin(\tan^{-1}x)+\sin(\cot^{-1}x))^2-1,\ |x|>1$. If $\frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}(\sin^{-1}(f(x)))$ and $y(\sqrt{3})=\frac{\pi}{6}$, then $y(-\sqrt{3})$ is equal to: **[Jan. 8, 2020 (I)]**

(a) $\frac{2\pi}{3}$ (b) $-\frac{\pi}{6}$

(c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{3}$

26. Let $y=y(x)$ be a solution of the differential equation,

$$\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0, |x|<1.$$

If $y\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to: **[Jan. 8, 2020 (I)]**

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{\sqrt{3}}{2}$

27. If $y=y(x)$ is the solution of the differential equation, $e^y=e^x$ such that $y(0)=0$, then $y(1)$ is equal to: **[Jan. 7, 2020 (I)]**

(a) $1+\log_e 2$ (b) $2+\log_e 2$

(c) $2e$ (d) $\log_e 2$

28. The general solution of the differential equation $(y^2-x^3)\,dx-xydy=0\ (x\neq 0)$ is : **[April 12, 2019 (II)]**

(a) $y^2-2x^2+cx^3=0$ (b) $y^2+2x^3+cx^2=0$

(c) $y^2+2x^2+cx^3=0$ (d) $y^2-2x^3+cx^2=0$

(where $c$ is a constant of integration)

**29.** If $\cos x\frac{dy}{dx}-y\sin x=6x,(0<x<\frac{\pi}{2})$ and $y\left(\frac{\pi}{3}\right)=0,$ then $y\left(\frac{\pi}{6}\right)$ is equal to: **[April. 09, 2019 (II)]**

(a) $\frac{\pi^2}{2\sqrt{3}}$ (b) $-\frac{\pi^2}{2}$

(c) $-\frac{\pi^2}{2\sqrt{3}}$ (d) $-\frac{\pi^2}{4\sqrt{3}}$

**30.** Given that the slope of the tangent to a curve $y=y(x)$ at any point $(x, y)$ is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2+y^2-2x-2y=0$, then its equation is: **[April. 08, 2019 (II)]**

(a) $x\log_e|y|=2(x-1)$

(b) $x\log_e|y|=-2(x-1)$

(c) $x^2\log_e|y|=-2(x-1)$

(d) $x\log_e|y|=x-1$

**31.** The solution of the differential equation, $\frac{dy}{dx}=(x-y)^2$, when $y(1)=1$, is: **[Jan. 11, 2019 (II)]**

(a) $\log_e\left|\frac{2-x}{2-y}\right|=x-y$

(b) $-\log_e\left|\frac{1-x+y}{1+x-y}\right|=2(x-1)$

(c) $-\log_e\left|\frac{1+x-y}{1-x+y}\right|=x+y-2$

(d) $\log_e\left|\frac{2-y}{2-x}\right|=2(y-1)$

**32.** If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals: **[10 Jan 2019 I]**

(a) $\frac{1}{3}+e^6$ (b) $\frac{1}{3}$

(c) $-\frac{4}{3}$ (d) $\frac{1}{3}+e^3$

**33.** The curve amongst the family of curves represented by the differential equation, $(x^2-y^2)\,dx+2xydy=0$ which passes through (1, 1) is: **[Jan. 10, 2019 (II)]**

(a) a circle with centre on the $x$-axis.

(b) an ellipse with major axis along the $y$-axis.

(c) a circle with centre on the $y$-axis.

(d) a hyperbola with transverse axis along the $x$-axis.

**34.** Let $f$ : [0, 1] $\to$ R be such that $f(xy)=f(x).f(y)$, for all $x, y\in[0, 1]$, and $f(0)\neq 0$. If $y=y(x)$ satisfies the differential equation, $\frac{dy}{dx}=f(x)$ with $y(0)=1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to: **[Jan. 09, 2019 (II)]**

(a) 3 (b) 4

(c) 2 (d) 5

**35.** The curve satisfying the differential equation, $(x^2-y^2)\,dx+2xydy=0$ and passing through the point (1, 1) is **[Online April 15, 2018]**

(a) a circle of radius two (b) a circle of radius one

(c) a hyprbola (d) an ellipse

**36.** If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is equal to : **[2017]**

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$

(c) $-\frac{2}{3}$ (d) $-\frac{1}{3}$

**37.** If a curve y = f(x) passes through the point (1, –1) and satisfies the differential equation, $y(1+xy)\,dx=x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to : **[2016]**

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $-\frac{2}{5}$ (d) $-\frac{4}{5}$

**38.** If f(x) is a differentiable function in the interval $((0,\infty)$ such that f(a) = 1 and $\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x}=1$, for each x > 0, then $f\left(\frac{3}{2}\right)$ is equal to : **[Online April 9, 2016]**

(a) $\frac{23}{18}$ (b) $\frac{13}{6}$

(c) $\frac{25}{9}$ (d) $\frac{31}{18}$

**39.** The solution of the differential equation $ydx-(x+2y^2)dy=0$ is $x=f(y)$. If f(–1) = 1, then f(a) is equal to : **[Online April 11, 2015]**

(a) 4 (b) 3

(c) 1 (d) 2

**40.** If y($x$) is the solution of the differential equation

$$(x+2)\frac{dy}{dx}=x^2+4x-9, x\neq -2 \text{ and } y(0)=0, \text{ then } y(-4)$$

is equal to : **[Online April 10, 2015]**

(a) 0 (b) 2

(c) 1 (d) –1

**41.** Let the population of rabbits surviving at time $t$ be governed by the differential equation $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200.$ If $p(0)=100$, then $p(t)$ equals: **[2014]**

(a) $600-500\,e^{t/2}$ (b) $400-300\,e^{-t/2}$

(c) $400-300\,e^{t/2}$ (d) $300-200\,e^{-t/2}$

**42.** If the general solution of the differential equation $y'=\frac{y}{x}+\Phi\left(\frac{x}{y}\right)$, for some function $\Phi$, is given by $y\,ln\,|cx|=x$, where c is an arbitrary constant, then $\Phi(2)$ is equal to: **[Online April 11, 2014]**

(a) 4 (b) $\frac{1}{4}$

(c) $-4$ (d) $-\frac{1}{4}$

**43.** At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers $x$ is given by $\frac{dP}{dx}=100-12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is **[2013]**

(a) 2500 (b) 3000

(c) 3500 (d) 4500

**44.** If a curve passes through the point $\left(2,\frac{7}{2}\right)$ and has slope $\left(1-\frac{1}{x^2}\right)$ at any point $(x, y)$ on it, then the ordinate of the point on the curve whose abscissa is – 2 is : **[Online April 23, 2013]**

(a) $-\frac{3}{2}$ (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) $-\frac{5}{2}$

**45.** Consider the differential equation :

$$\frac{dy}{dx}=\frac{y^3}{2\left(xy^2-x^2\right)}$$

**[Online April 22, 2013]**

**Statement-1:** The substitution $z=y^2$ transforms the above equation into a first order homogenous differential equation.

**Statement-2:** The solution of this differential equation is $y^2e^{-y^2/x}=C$.

(a) Both statements are false.

(b) Statement-1 is true and statement-2 is false.

(c) Statement-1 is false and statement-2 is true.

(d) Both statements are true.

**46.** The population $p$ ($t$) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt}=0.5\,p(t)-450$. If $p$ (0) = 850, then the time at which the population becomes zero is : **[2012]**

(a) $2ln\,18$ (b) $ln\,9$

(c) $\frac{1}{2}ln\,18$ (d) $ln\,18$

**47.** Let $y(x)$ be a solution of $\frac{(2+\sin x)}{(1+y)}\frac{dy}{dx}=\cos x$. If $y(0)=2$, then $y\left(\frac{\pi}{2}\right)$ equals **[Online May 7, 2012]**

(a) $\frac{5}{2}$ (b) 2

(c) $\frac{7}{2}$ (d) 3

**48.** The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact is given by : **[2011RS]**

(a) $2y-3x=0$ (b) $y=\frac{6}{x}$

(c) $x^2+y^2=13$ (d) $\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2=2$

**49.** Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for $t$ years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt}=-k(T-t)$, where $k>0$ is a constant and $T$ is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is **[2011]**

(a) $I-\frac{kT^2}{2}$ (b) $I-\frac{k(T-t)^2}{2}$

**50.** If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to : **[2011]**

(a) 5 (b) 13
(c) $-2$ (d) 7

**51.** The solution of the differential equation $\frac{dy}{dy} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is **[2008]**

(a) $y = \ln x + x$ (b) $y = x\ln x + x^2$
(c) $y = xe^{(x-1)}$ (d) $y = x\ln x + x$

**52.** The normal to a curve at $P(x, y)$ meets the $x$-axis at $G$. If the distance of $G$ from the origin is twice the abscissa of $P$, then the curve is a **[2007]**

(a) circle (b) hyperbola
(c) ellipse (d) parabola.

**53.** If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is **[2005]**

(a) $y\log\left(\frac{x}{y}\right) = cx$ (b) $x\log\left(\frac{y}{x}\right) = cy$
(c) $\log\left(\frac{y}{x}\right) = cx$ (d) $\log\left(\frac{x}{y}\right) = cy$

**54.** The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ **[2002]**

(a) $\frac{e^{-2x}}{4}$ (b) $\frac{e^{-2x}}{4} + cx + d$
(c) $\frac{1}{4}e^{-2x} + cx^2 + d$ (d) $\frac{1}{4}e^{-4x} + cx + d$

## TOPIC 3 Linear Differential Equation of First Order Different Equation of the form: $\frac{d^2y}{dx^2} = F(x)$, Solution by Inspection Method

**55.** Let $y = y(x)$ be the solution of the differential equation $\cos x\frac{dy}{dx} + 2y\sin x = \sin 2x,\ x \in \left(0, \frac{\pi}{2}\right)$.
If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to : **[Sep. 05, 2020 (II)]**

(a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$
(c) $\sqrt{2} - 2$ (d) $\frac{1}{\sqrt{2}} - 1$

**56.** Let $y = y(x)$ be the solution of the differential equation, $xy' - y = x^2(x\cos x + \sin x),\ x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to : **[Sep. 04, 2020 (I)]**

(a) $2 + \frac{\pi}{2}$ (b) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$
(c) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$ (d) $1 + \frac{\pi}{2}$

**57.** If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation, $(x+1)dy = ((x+1)^2 + y - 3)dx,\ y(2) = 0$, then $y(3)$ is equal to _____. **[NA Jan. 09, 2020 (I)]**

**58.** Let $y = y(x)$ be the solution curve of the differential equation, $\left(y^2 - x\right)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the $x$-axis at a point whose abscissa is: **[Jan. 7, 2020 (II)]**

(a) $2 - e$ (b) $-e$
(c) 2 (d) $2 + e$

**59.** Consider the differential equation, $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. If value of $y$ is 1 when $x = 1$, then the value of $x$ for which $y = 2$, is : **[April 12, 2019 (I)]**

(a) $\frac{5}{2} + \frac{1}{\sqrt{e}}$ (b) $\frac{3}{2} - \frac{1}{\sqrt{e}}$
(c) $\frac{1}{2} + \frac{1}{\sqrt{e}}$ (d) $\frac{3}{2} - \sqrt{e}$

**60.** If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to : **[April 10, 2019 (I)]**

(a) $e - 2$ (b) $\frac{1}{2} - e$
(c) $2 + \frac{1}{e}$ (d) $\frac{1}{e} - 2$

**61.** Let $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx}+y\tan x = 2x+x^2\tan x$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that y (0) = 1. Then : **[April 10, 2019 (II)]**

(a) $y\left(\frac{\pi}{4}\right)+y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{2}+2$

(b) $y'\left(\frac{\pi}{4}\right)+y'\left(-\frac{\pi}{4}\right)=-\sqrt{2}$

(c) $y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=\sqrt{2}$

(d) $y'\left(\frac{\pi}{4}\right)-y'\left(-\frac{\pi}{4}\right)=\pi-\sqrt{2}$

**62.** The solution of the differential equation $x\frac{dy}{dx}+2y=x^2$ $(x \neq 0)$ with y(1) = 1, is: **[April 09, 2019 (I)]**

(a) $y=\frac{4}{5}x^3+\frac{1}{5x^2}$ (b) $y=\frac{x^3}{5}+\frac{1}{5x^2}$

(c) $y=\frac{x^2}{4}+\frac{3}{4x^2}$ (d) $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

**63.** Let $y = y(x)$ be the solution of the differential equation, $(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y=1$ such that $y(0) = 0$. If $\sqrt{a}\,y(1)=\frac{\pi}{32}$, then the value of '$a$' is : **[April 08, 2019 (I)]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) $\frac{1}{16}$

**64.** Let y = y(x) be the solution of the differential equation, $x\frac{dy}{dx}+y=x\log_e x, (x>1)$. If $2y(2)=\log_e 4-1$, then y(e) is equal to : **[Jan. 12, 2019 (I)]**

(a) $-\frac{e}{2}$ (b) $-\frac{e^2}{2}$

(c) $\frac{e}{4}$ (d) $\frac{e^2}{4}$

**65.** If a curve passes through the point (1, – 2) and has slope of the tangent at any point $(x, y)$ on it as $\frac{x^2-2y}{x}$, then the curve also passes through the point :**[Jan. 12, 2019 (II)]**

(a) (3, 0) (b) $(\sqrt{3},0)$

(c) (–1, 2) (d) $(-\sqrt{2},1)$

**66.** If y(x) is the solution of the differential equation $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}, x>0$, where $y(1)=\frac{1}{2}e^{-2}$, then : **[Jan. 11, 2019 (I)]**

(a) $y(\log_e 2)=\log_e 4$

(b) $y(\log_e 2)=\frac{\log_e 2}{4}$

(c) $y(x)$ is decreasing in $\left(\frac{1}{2},1\right)$

(d) y (x) is decreasing in (0, 1)

**67.** Let $f$ be a differentiable function such that $f'(x)=7-\frac{3}{4}\frac{f(x)}{x}$, $(x>0)$ and $f(1)\neq 4$. Then $\lim_{x\to 0^+} x f\left(\frac{1}{x}\right)$: **[Jan. 10, 2019 (II)]**

(a) exists and equals $\frac{4}{7}$. (b) exists and equals 4.

(c) does not exist. (d) exists and equals 0.

**68.** If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx}+2y=x^2$ satisfying $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to: **[Jan. 09, 2019 (I)]**

(a) $\frac{7}{64}$ (b) $\frac{1}{4}$

(c) $\frac{49}{16}$ (d) $\frac{13}{16}$

**69.** Let y – y(x) be the solution of the differential equation $\sin x\frac{dy}{dx}+y\cos x=4x, x\in(0,\pi)$. If $y\left(\frac{\pi}{2}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to : **[2018]**

(a) $\frac{-8}{9\sqrt{3}}\pi^2$ (b) $-\frac{8}{9}\pi^2$

(c) $-\frac{4}{9}\pi^2$ (d) $\frac{4}{9\sqrt{3}}\pi^2$

**70.** Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx}+2y=f(x)$, where $f(x)=\begin{cases}1, & x\in[0,1]\\ 0, & otherwise\end{cases}$

If $y(0) = 0$, then $y\left(\frac{3}{2}\right)$ is **[Online April 15, 2018]**

(a) $\frac{e^2-1}{2e^3}$ (b) $\frac{e^2-1}{e^3}$

(c) $\frac{1}{2e}$ (d) $\frac{e^2+1}{2e^4}$

**71.** The curve satisfying the differential equation, $ydx-(x+3y^2)\,dy=0$ and passing through the point (1, 1), also passes through the point : **[Online April 8, 2017]**

(a) $\left(\frac{1}{4},-\frac{1}{2}\right)$ (b) $\left(-\frac{1}{3},\frac{1}{3}\right)$

(c) $\left(\frac{1}{3},-\frac{1}{3}\right)$ (d) $\left(\frac{1}{4},\frac{1}{2}\right)$

**72.** The solution of the differential equaiton $\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}$, where $0\le x<\frac{\pi}{2}$, and $y(0)=1$, is given by : **[Online April 10, 2016]**

(a) $y^2=1+\frac{x}{\sec x+\tan x}$ (b) $y=1+\frac{x}{\sec x+\tan x}$

(c) $y=1-\frac{x}{\sec x+\tan x}$ (d) $y^2=1-\frac{x}{\sec x+\tan x}$

**73.** Let y(x) be the solution of the differential equation $(x\log x)\frac{dy}{dx}+y=2x\log x,(x\ge 1)$. Then y (e) is equal to: **[2015]**

(a) 2 (b) 2e

(c) e (d) 0

**74.** If $\frac{dy}{dx}+y\tan x=\sin 2x$ and $y(0)=1$, then $y(\pi)$ is equal to: **[Online April 19, 2014]**

(a) 1 (b) $-1$

(c) $-5$ (d) 5

**75.** The general solution of the differential equation, $\sin 2x\left(\frac{dy}{dx}-\sqrt{\tan x}\right)-y=0$, is : **[Online April 12, 2014]**

(a) $y\sqrt{\tan x}=x+c$ (b) $y\sqrt{\cot x}=\tan x+c$

(c) $y\sqrt{\tan x}=\cot x+c$ (d) $y\sqrt{\cot x}=x+c$

**76.** The equation of the curve passing through the origin and satisfying the differential equation $(1+x^2)\frac{dy}{dx}+2xy=4x^2$ is **[Online April 25, 2013]**

(a) $(1+x^2)y=x^3$ (b) $3(1+x^2)y=2x^3$

(c) $(1+x^2)y=3x^3$ (d) $3(1+x^2)y=4x^3$

**77.** The integrating factor of the differential equation $\left(x^2-1\right)\frac{dy}{dx}+2xy=x$ is **[Online May 26, 2012]**

(a) $\frac{1}{x^2-1}$ (b) $x^2-1$

(c) $\frac{x^2-1}{x}$ (d) $\frac{x}{x^2-1}$

**78.** The general solution of the differential equation $\frac{dy}{dx}+\frac{2}{x}y=x^2$ is **[Online May 19, 2012]**

(a) $y=cx^{-3}-\frac{x^2}{4}$ (b) $y=cx^3-\frac{x^2}{4}$

(c) $y=cx^2+\frac{x^3}{5}$ (d) $y=cx^{-2}+\frac{x^3}{5}$

**79.** Consider the differential equation **[2011RS]**

$y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If $y(1)=1$, then $x$ is given by :

(a) $4-\frac{2}{y}-\frac{e^{\frac{1}{y}}}{e}$ (b) $3-\frac{1}{y}+\frac{e^{\frac{1}{y}}}{e}$

(c) $1+\frac{1}{y}-\frac{e^{\frac{1}{y}}}{e}$ (d) $1-\frac{1}{y}+\frac{e^{\frac{1}{y}}}{e}$

**80.** Solution of the differential equation $\cos x\,dy=y(\sin x-y)\,dx,\ 0<x<\frac{\pi}{2}$ is **[2010]**

(a) $y\sec x=\tan x+c$

(b) $y\tan x=\sec x+c$

(c) $\tan x=(\sec x+c)y$

(d) $\sec x=(\tan x+c)y$

**81.** Solution of the differential equation $ydx+(x+x^2y)dy=0$ is **[2004]**

(a) $\log y=Cx$ (b) $-\frac{1}{xy}+\log y=C$

(c) $\frac{1}{xy}+\log y=C$ (d) $-\frac{1}{xy}=C$

**82.** The solution of the differential equation $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$, is **[2003]**

(a) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$

(b) $(x-2)=ke^{2\tan^{-1}y}$

(c) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

(d) $xe^{\tan^{-1}y}=\tan^{-1}y+k$

# Hints & Solutions

**1.** **(a)** Since, $x^2 = 4b(y + b)$

$x^2 = 4by + 4b^2$

$2x = 4by'$

$\Rightarrow \quad b = \dfrac{x}{2y'}$

So, differential equation is

$$x^2 = \frac{2x}{y'}.y + \left(\frac{x}{y'}\right)^2$$

$x(y')^2 = 2yy' + x$

**2.** **(c)** $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

Since, it passes through (0, 3)

$\therefore \dfrac{0}{a^2} + \dfrac{9}{b^2} = 1$

$\Rightarrow b^2 = 9$

$\therefore$ eq. of ellipse becomes:

$$\frac{x^2}{a^2} + \frac{y^2}{9} = 1$$

differential w.r.t.$x$, we get;

$$\frac{2x}{a^2} + \frac{2y}{9}\frac{dy}{dx} = 0$$

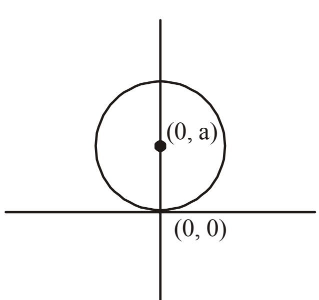
$$\Rightarrow \frac{y}{x}\left(\frac{dy}{dx}\right) = \frac{-9}{a^2}$$

Again differentiating w.r.t.$x$, we get;

$$\frac{y}{x}\frac{d^2y}{dx^2} + \frac{x\frac{dy}{dx} - y}{x^2}\frac{dy}{dx} = 0$$

$\Rightarrow xyy'' + x(y')^2 - yy' = 0$

**3.** **(c)** Since family of all circles touching $x$-axis at the origin




$\therefore$ Eqn is $(x)^2 + (y - a)^2 = a^2$

where $(0, a)$ is the centre of circle.

$$\Rightarrow x^2 + y^2 + a^2 - 2ay = a^2$$

$$\Rightarrow x^2 + y^2 - 2ay = 0 \quad ...(1)$$

Differentiate both side w.r.t '$x$', we get

$$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0$$

$$\Rightarrow x + y\frac{dy}{dx} - a\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}} = a$$

Put value of '$a$' in eqn (1), we get

$$x^2 + y^2 - 2y\left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right] = 0$$

$$\Rightarrow (x^2 + y^2)\frac{dy}{dx} - 2y^2\frac{dy}{dx} - 2xy = 0$$

$$\Rightarrow (x^2 + y^2 - 2y^2)\frac{dy}{dx} = 2xy$$

$$\Rightarrow (x^2 - y^2)\frac{dy}{dx} = 2xy \equiv g(x)y$$

Hence, $g(x) = 2x$

**4.** **(b)** Statement -1 : $y^2 = \pm 4ax$

$$\Rightarrow \frac{dy}{dx} = \pm 2a.\frac{1}{y} \Rightarrow \frac{dy}{dx} \propto \frac{1}{y}$$

Statement -2 : $y^2 = 4ax \Rightarrow 2y\dfrac{dy}{dx} = 4a$

Thus both statements are true but statement-2 is not a correct explanation for statement-1.

**5.** **(d)** **Statement - 1**

Given differential equations are $\dfrac{dy}{dx} + y^2 = x$ and

$$\frac{d^2y}{dx^2} + y = \sin x$$

Their degrees are 1.

Both have equal degree.

Also, Statement - 2 is the correct explanation for Statement

**6.** **(c)** We have $y = c_1 e^{c_2 x}$

Differentiate it w.r. to $x$

$$\Rightarrow \quad y' = c_1 c_2 e^{c_2 x} = c_2 y$$

$$\Rightarrow \quad \frac{y'}{y} = c_2 \text{ Differentiate it w.r. to } x$$

$$\Rightarrow \quad \frac{y'' y - (y')^2}{y^2} = 0 \quad \Rightarrow \quad y'' y = (y')^2$$

**7.** **(c)** Let the centre of the circle be $(h, 2)$

$\therefore$ Equation of circle is

$$(x-h)^2 + (y-2)^2 = 25 \qquad \ldots(1)$$

Differentiating with respect to $x$, we get

$$2(x-h) + 2(y-2)\frac{dy}{dx} = 0$$

$$\Rightarrow x - h = -(y-2)\frac{dy}{dx}$$

Substituting in equation (1) we get

$$(y-2)^2\left(\frac{dy}{dx}\right)^2 + (y-2)^2 = 25$$

$$\Rightarrow (y-2)^2 (y')^2 = 25 - (y-2)^2$$

**8.** **(a)** General equation of circles passing through origin and having their centres on the x-axis is

$$x^2 + y^2 + 2gx = 0 \qquad \ldots(i)$$

On differentiating w.r.t $x$, we get

$$2x + 2y \cdot \frac{dy}{dx} + 2g = 0 \Rightarrow g = -\left(x + y\frac{dy}{dx}\right)$$

Putting in (i)

$$x^2 + y^2 + 2\left\{-\left(x + y\frac{dy}{dx}\right)\right\} . x = 0$$

$$\Rightarrow \ x^2 + y^2 - 2x^2 - 2x\frac{dy}{dx} . y = 0$$

$$\Rightarrow \ y^2 = x^2 + 2xy\frac{dy}{dx}$$

**9.** **(d)** $Ax^2 + By^2 = 1 \qquad \ldots(i)$

Differentiate w.r. to $x$

$$Ax + By\frac{dy}{dx} = 0 \qquad \ldots(ii)$$

Again differentiate w.r. to $x$

$$A + By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0 \qquad \ldots(iii)$$

From (ii) and (iii)

$$x\left\{-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right\} + By\frac{dy}{dx} = 0$$

Dividing both sides by –B, we get

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

Therefore order 2 and degree 1

**10.** **(c)** $y^2 = 2c(x + \sqrt{c}) \qquad \ldots\ldots\ldots (i)$

Differentite it w.r. to $x$

$2yy' = 2c.1$ or $yy' = c \qquad \ldots\ldots\ldots (ii)$

[On putting value of $c$ from (ii) in (i)]

$$\Rightarrow \quad y^2 = 2yy'(x + \sqrt{yy'})$$

On simplifying, we get

$$(y - 2xy')^2 = 4yy'^3 \qquad \ldots\ldots\ldots (iii)$$

Hence equation (iii) is of order 1 and degree 3.

**11.** **(c)** $x^2 + y^2 - 2ay = 0 \qquad \ldots\ldots(1)$

Differentiate w.r. to $x$,

$$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0 \ \Rightarrow a = \frac{x + yy'}{y'}$$

Putting in (1) we get, $x^2 + y^2 - 2\left(\dfrac{x + yy'}{y'}\right)y = 0$

$$\Rightarrow (x^2 + y^2)y' - 2xy - 2y^2 y' = 0$$

$$\Rightarrow (x^2 - y^2)y' = 2xy$$

**12.** **(c)** $y^2 = 4a(x - h)$,

Differentiating $2yy_1 = 4a \Rightarrow yy_1 = 2a$

Again differentiating, we get

$$\Rightarrow y_1^2 + yy_2 = 0$$

Degree $= 1$, order $= 2$.

**13.** **(c)** $\left(1 + 3\dfrac{dy}{dx}\right)^2 = \left(\dfrac{4d^3y}{dx^3}\right)^3$

$$\Rightarrow \left(1 + 3\frac{dy}{dx}\right)^2 = 16\left(\frac{d^3y}{dx^3}\right)^3$$

Order $= 3$, degree 3

**14.** **(a)** $\sqrt{1+x^2} \cdot \sqrt{1+y^2} = -xy\dfrac{dy}{dx}$

$$\int \frac{\sqrt{1+x^2}}{x}dx = -\int \frac{y}{\sqrt{1+y^2}}dy$$

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

$$\Rightarrow \int \frac{\sec^3\theta d\theta}{\tan\theta} = -\int \frac{2y}{2\sqrt{1+y^2}}dy$$

$$\Rightarrow \int \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos^2\theta}d\theta = -\sqrt{1+y^2}$$

$$\Rightarrow \int (\tan\theta \cdot \sec\theta + \text{cosec}\theta)d\theta = -\sqrt{1+y^2}$$

$$\Rightarrow \sec\theta + \log_e|\text{cosec}\theta - \cot\theta| = -\sqrt{1+y^2} + C$$

$$\therefore \sqrt{1+x^2} + \log\left|\frac{\sqrt{1+x^2} - 1}{\ldots}\right| = -\sqrt{1+y^2} + C$$

$$\Rightarrow \sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$$

**15. (a)** $\because y=\left(\frac{2}{\pi}x-1\right)\operatorname{cosec}x$

$$\frac{dy}{dx}=\frac{2}{\pi}\operatorname{cosec}x-\left(\frac{2}{\pi}x-1\right)\operatorname{cosec}x\cdot\cot x$$

$$=\operatorname{cosec}x\left[\frac{2}{\pi}-\left(\frac{2}{\pi}x-1\right)\cot x\right]$$

$$\Rightarrow \frac{dy}{dx}-\frac{2}{\pi}\operatorname{cosec}x=y\cot x \qquad \text{...(i)}$$

It is given that,

$$\Rightarrow \frac{dy}{dx}-\frac{2}{\pi}\operatorname{cosec}x=-yp(x) \qquad \text{...(ii)}$$

By comparision of (i) and (ii), we get

$p(x)=\cot x$

**16. (b)** $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx}=-e^x$

$$\int\frac{dy}{2+y}=-\int\frac{e^x}{5+e^x}dx$$

$$\Rightarrow \log_e|2+y|\cdot\log_e|5+e^x|=\log_e C$$

$$\Rightarrow |(2+y)(5+e^x)|=C \qquad \because y(0)=1$$

$C=18.$

$$\therefore (2+y)\cdot(5+e^x)=18$$

When $x=\log_e 13$ then $(2+y)\cdot 18=18$

$$\Rightarrow 2+y=\pm 1$$

$$\therefore y=-1,-3$$

$$\therefore y(\ln 13)=-1$$

**17. (a)** Let $y+3x=t$

$$\Rightarrow \frac{dy}{dx}+3=\frac{dt}{dx}$$

Putting these value in given differential equation

$$\frac{dt}{dx}=\frac{t}{\log_e t}$$

$$\Rightarrow \int\frac{\log_e t}{t}dt=\int dx$$

$$\Rightarrow \frac{(\log_e t)^2}{2}=x-C$$

$$\Rightarrow x-\frac{1}{2}(\ln(y+3x))^2=C$$

**18. (a)** $\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$

$$\Rightarrow \lim_{t\to x}\frac{2tf^2(x)-2x^2f(t)\cdot f'(t)}{1}=0$$

Using L'Hospital's rule

$$\Rightarrow f(x)=xf'(x)$$

$$\int\frac{f'(x)}{f(x)}dx=\int\frac{1}{x}dx$$

$$\log_e f(x)=\log_e x+\log_e C$$

$\Rightarrow f(x)=Cx, \qquad \because f(1)=e$

$\Rightarrow C=e;$ so $f(x)=ex$

When $f(x)=1=ex\Rightarrow x=\frac{1}{e}$

**19. (c)** $\int\left(\frac{y^2+1}{y^2}\right)dy=\int\frac{e^x dx}{e^x+1}$

$$\Rightarrow y-\frac{1}{y}=\log_e|e^x+1|+c$$

$\because$ Passes through (0, 1).

$\therefore c=-\log_e 2$

$$\Rightarrow y^2-1=y\log_e\left(\frac{e^x+1}{2}\right)$$

$$\Rightarrow y^2=1+y\log_e\left(\frac{e^x+1}{2}\right)$$

**20. (b)** $x^3dy+xy\,dyx=2y\,dx+x^2\,dy$

$$\Rightarrow (x^3-x^2)dy=(2-x)y\,dx$$

$$\Rightarrow \frac{dy}{y}=\frac{2-x}{x^2(x-1)}dx$$

$$\Rightarrow \int\frac{dy}{y}=\int\frac{2-x}{x^2(x-1)}dx \qquad \text{...(i)}$$

Let $\frac{2-x}{x^2(x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$

$$\Rightarrow 2-x=A(x-1)+B(x-1)+Cx^2$$

Compare the coefficients of $x$, $x^2$ and constant term.

$C=1, B=-2$ and $A=-1$

$$\therefore \int \frac{dy}{y} = \int \left\{ \frac{-1}{x} - \frac{2}{x^2} + \frac{1}{x-1} \right\} dx$$

$$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln|x-1| + C$$

$$\therefore y(2) = e$$

$$\Rightarrow 1 = -\ln 2 + 1 + 0 + C \qquad [\because \log e = 1]$$

$$\Rightarrow C = \ln 2$$

$$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln|x-1| + \ln 2$$

At $x = 4$,

$$\Rightarrow \ln y(4) = -\ln 4 + \frac{1}{2} + \ln 3 + \ln 2$$

$$\Rightarrow \ln y(4) = \ln\left(\frac{3}{2}\right) + \frac{1}{2} = \ln\left(\frac{3}{2}e^{1/2}\right)$$

$$[\because \log m + \log n = \log(mn)]$$

$$\Rightarrow y(4) = \frac{3}{2}e^{1/2}$$

**21.** **(c)** The given differential equation is

$$\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x,\ y > 0$$

$$\Rightarrow \frac{dy}{y+1} = -\frac{\cos x}{2+\sin x}dx$$

Integrate both sides,

$$\int \frac{dy}{y+1} = \int \frac{(-\cos x)dx}{2+\sin x}$$

$$\ln|y+1| = -\ln|2+\sin x| + \ln C$$

$$\Rightarrow \ln|y+1| + \ln|2+\sin x| = \ln C$$

$$\Rightarrow \ln|(y+1)(2+\sin x)| = \ln C$$

$$\because y(0) = 1 \Rightarrow \ln 4 = \ln C \Rightarrow C = 4$$

$$\therefore (y+1)(2+\sin x) = 4$$

$$\Rightarrow y = \frac{4}{2+\sin x} - 1$$

$$\therefore y = \frac{2-\sin x}{2+\sin x} \Rightarrow y(\pi) = \frac{2-\sin\pi}{2+\sin\pi} = 1$$

$$\Rightarrow a = 1$$

Now, $\dfrac{dy}{dx} = \dfrac{(2+\sin x)(-\cos x) - (2-\sin x)\cdot\cos x}{(2+\sin x)^2}$

$$\left.\frac{dy}{dx}\right|_{x=\pi} = 1 \Rightarrow b = 1.$$

Ordered pair $(a, b) = (1, 1)$.

**22.** **(a)** $\dfrac{dy}{dx} = \dfrac{2xy + y^2}{2x^2}$

It is homogeneous differential equation.

$\therefore$ Put $y = vx$

$$\Rightarrow v + x\frac{dv}{dx} = v + \frac{v^2}{2} \Rightarrow \int 2\frac{dv}{v^2} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{-2}{v} = \log_e x + c \Rightarrow \frac{-2x}{y} = \log_e x + c$$

Put $x = 1, y = 2$, we get $c = -1$

$$\Rightarrow \frac{-2x}{y} = \log_e x - 1$$

Hence, put $x = \dfrac{1}{2} \Rightarrow y = \dfrac{1}{1+\log_e 2}$

**23.** **(a)** $f'(x) = \tan^{-1}(\sec x + \tan x)$

$$= \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right) = \frac{\pi}{4}+\frac{x}{2}$$

Integrate both sides, we get

$$\int (f'(x))\,dx = \int\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$

$$f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + C$$

$\because \quad f(0) = 0$

$$C = 0 \Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$$

So, $f(1) = \dfrac{\pi+1}{4}$

**24.** **(d)** The given differential equation,

$$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Then, $v + x\frac{dv}{dx} = \frac{vx^2}{x^2 + v^2x^2} = \frac{v}{1+v^2}$

$\Rightarrow \frac{1+v^2}{v^3}dv = -\frac{1}{x}dx$

$\Rightarrow \int\left(\frac{1}{v^3} + \frac{1}{v}\right)dv = \int\frac{-1}{x}dx$

$\Rightarrow \frac{-1}{2}\left(\frac{1}{v^2}\right) + \ln v = -\ln x + c$

$\Rightarrow -\frac{x^2}{2y^2} = -\ln y + c \qquad \left[\because v = \frac{y}{x}\right]$

When $x = 1, y = 1$, then $-\frac{1}{2} = c$

$\Rightarrow x^2 = y^2(1 + 2\ln y)$

At $y = e, x^2 = e^2(3)$

$\Rightarrow x = \pm\sqrt{3}e$

So, $x = \sqrt{3}e$

**25.** **(b)** $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1} f(x)\right)$

$2y = \sin^{-1} f(x) + C = \sin^{-1}(\sin(2\tan^{-1}x)) + C$

$\Rightarrow 2\left(\frac{\pi}{6}\right) = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + C$

$\frac{\pi}{3} = \frac{\pi}{3} + C \quad \therefore C = 0$

for $x = -\sqrt{3}$, $2y = \sin^{-1}\left(\sin\left(\frac{-2\pi}{6}\right)\right) + 0$

$\Rightarrow 2y = \frac{-\pi}{3} \Rightarrow y = \frac{-\pi}{6}$

**26.** **(c)** The given differential eqn. is

$\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0 \Rightarrow \sin^{-1}y + \sin^{-1}x = c$

At $x = \frac{1}{2}, y = \frac{\sqrt{3}}{2} \Rightarrow c = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}y = \cos^{-1}x$

Hence, $y\left(-\frac{1}{\sqrt{2}}\right) = \sin\left(\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)$

$= \sin\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) = \frac{1}{\sqrt{2}}$

**27.** **(a)** Let $e^y = t$

$e^y\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \frac{dt}{dx} - t = e^x \qquad \left[\because e^y\frac{dy}{dx} - e^y = e^x\right]$

$\text{I.F.} = e^{\int -1.dx} = e^{-x}$

$t(e^{-x}) = \int e^x.e^{-x}dx \Rightarrow e^{y-x} = x + c$

Put $x = 0, y = 0$, then we get $c = 1$

$e^{y-x} = x + 1$

$y = x + \log_e(x+1)$

Put $x = 1 \ \therefore\ y = 1 + \log_e 2$

**28.** **(b)** Given differential equation can be written as,

$y^2dx - xydy = x^3dx$

$\Rightarrow \frac{(ydx - xdy)y}{x^2} = xdx \Rightarrow -yd\left(\frac{y}{x}\right) = xdx$

$\Rightarrow -\frac{y}{x}.d\left(\frac{y}{x}\right) = dx \Rightarrow -\frac{1}{2}\left(\frac{y}{x}\right)^2 = x + c_1$

$\Rightarrow 2x^3 + cx^2 + y^2 = 0 \qquad [\text{Here, } c = 2c_1]$

**29.** **(c)** $\cos x dy - (\sin x)y\,dx = 6x\,dx$

$\Rightarrow \int d(y\cos x) = \int 6x\,dx \Rightarrow y\cos x = 3x^2 + C$ ...(1)

Given, $y\left(\frac{\pi}{3}\right) = 0$

Putting $x = \frac{\pi}{3}$ and $y = 0$ in eq. (1), we get

$(10)\times\left(\frac{1}{2}\right) = \frac{3\pi^2}{9} + C \Rightarrow C\frac{-\pi^2}{3}$

So, from (1) $y\cos x = 3x^2 - \frac{\pi^2}{3}$

Now, put $x = \frac{\pi}{6}$ in the above equation,

$y\frac{\sqrt{3}}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3} \Rightarrow \frac{\sqrt{3}y}{2} = \frac{-3\pi^2}{12} \Rightarrow y = \frac{-\pi^2}{2\sqrt{3}}$

**30.** **(1)** Given $\frac{dy}{dx} = \frac{2y}{x^2}$

Integrating both sides, $\int\frac{dy}{y} = 2\int\frac{dx}{x^2}$

$\Rightarrow \ln|y| = -\frac{2}{x} + C$ ...(i)

Equation (i) passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, i.e., (1, 1)

$\therefore\ C = 2$

Now, $\ln|y| = -\frac{2}{x} + 2$

$x\ln|y| = -2(1-x) \Rightarrow x\ln|y| = 2(x-1)$

**31. (b)** The given differential equation

$\frac{dy}{dx} = (x-y)^2 \quad \ldots(1)$

Let $x - y = t \Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

Now, from equation (1)

$\left(1 - \frac{dt}{dx}\right) = (t)^2$

$\Rightarrow 1 - t^2 = \frac{dt}{dx} \Rightarrow \int dx = \int \frac{dt}{1-t^2}$

$\Rightarrow -x = \frac{1}{2\times 1}\ln\left|\frac{t-1}{t+1}\right| + c$

$\Rightarrow -x = \frac{1}{2}\ln\left|\frac{x-y-1}{x-y+1}\right| + c$

$\because$ The given condition $y(1) = 1$

$-1 = \frac{1}{2}\ln\left|\frac{1-1-1}{1-1+1}\right| + c \Rightarrow c = -1$

Hence, $2(x-1) = -\ln\left|\frac{1-x+y}{1-y+x}\right|$

**32. (a)** Given, $\frac{dy}{dx} + \frac{3}{\cos^2 x}y = \frac{1}{\cos^2 x}$

$\frac{dy}{dx} = \sec^2 x(1-3y)$

$\Rightarrow \int \frac{dy}{(1-3y)} = \int \sec^2 x\, dx$

$\Rightarrow -\frac{1}{3}\ln|1-3y| = \tan x + C \ \ldots(i)$

$\because\ y\left(\frac{\pi}{4}\right) = \frac{4}{3}$ (Given)

$\Rightarrow -\frac{1}{3}\ln|1-4| = \tan\frac{\pi}{4} + C$

$\Rightarrow -\frac{1}{3}\ln 3 = C + 1 \Rightarrow C = -1 - \frac{1}{3}\ln 3$

$\therefore$ in eq. (i), we get

$\frac{-1}{3}\ln|1-3y| = \tan x - 1 - \frac{1}{3}\ln 3$

Put, $x = -\frac{\pi}{4}$

$\Rightarrow -\frac{1}{3}\ln|1-3y| = \tan\left(-\frac{\pi}{4}\right) - 1 - \frac{1}{3}\ln 3$

$= -1 - 1 - \frac{1}{3}\ln 3$

$\Rightarrow \ln|1-3y| = 6 + \ln 3$

$\Rightarrow \ln\left|\frac{1}{3} - y\right| = 6 \Rightarrow \left|\frac{1}{3} - y\right| = e^6 \Rightarrow y = \frac{1}{3} \pm e^6$

**33. (a)** $(x^2 - y^2)dx + 2xy\,dy = 0$

$y^2dx - 2xydy = x^2dx$

$2xydy - y^2dx = -x^2dx$

$d(xy^2) = -x^2\,dx$

$\frac{xd(y^2) - y^2d(x)}{x^2} = -dx$

$d\left(\frac{y^2}{x}\right) = -dx$

$\int d\left(\frac{y^2}{x}\right) = -\int dx$

$\frac{y^2}{x} = -x + C \quad \ldots(1)$

Since, the above curve passes through the point (1, 1)

Then, $\frac{1^2}{1} = -1 + C \Rightarrow C = 2$

Now, the curve (1) becomes

$y^2 = -x^2 + 2x$

$\Rightarrow y^2 = -(x-1)^2 + 1$

$(x-1)^2 + y^2 = 1$

The above equation represents a circle with centre (1, 0) and centre lies on $x$-axis.

**34. (a)** $f(xy) = f(x).f(y) \quad \ldots(1)$

Put $x = y = 0$ in (1) to get $f(0) = 1$

Put $x = y = 1$ in (1) to get $f(1) = 0$ or $f(1) = 1$

$f(1) = 0$ is rejected else $y = 1$ in (1) gives $f(x) = 0$ imply $f(0) = 0$.

Hence, $f(0) = 1$ and $f(1) = 1$

By first principle derivative formula,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0} f(x)\left(\frac{f\left(1+\frac{h}{x}\right)-f(1)}{h}\right)$$

$$\Rightarrow\ f'(x)=\frac{f(x)}{x}f'(1)$$

$$\Rightarrow\ \frac{f'(x)}{f(x)}=\frac{k}{x}\Rightarrow \ln f(x)=k\ln x+c$$

$f(1)=1\Rightarrow \ln 1=k\ln 1+c\Rightarrow c=0$

$\Rightarrow\ \ln f(x)=k\ln x\Rightarrow f(x)=x^k$ but $f(0)=1$

$\Rightarrow\ k=0$

$\therefore\ f(x)=1$

$$\frac{dy}{dx}=f(x)=1\Rightarrow y=x+c,\ y(0)=1\Rightarrow c=1$$

$\Rightarrow\ y=x+1$

$$\therefore\ y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)=\frac{1}{4}+1+\frac{3}{4}+1=3$$

**35.** **(b)** $(x^2-y^2)\,dx+2xydy=0$

$$\Rightarrow\ \frac{dy}{dx}=\frac{y^2-x^2}{2xy}$$

Let $y=vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\Rightarrow\ v+x\frac{dv}{dx}=\frac{v^2x^2-x^2}{2vx^2}\ \Rightarrow\ v+x\frac{dv}{dx}=\frac{v^2-1}{2v}$$

$$\Rightarrow\ x\frac{dv}{dx}=\frac{-v^2-1}{2v}$$

$$\Rightarrow\ \frac{2vdv}{v^2+1}=-\frac{dx}{x}$$

After integrating, we get

$ln\,|\,v^2+1|=-ln\,|\,x\,|+lnc$

$$\frac{y^2}{x^2}+1=\frac{c}{x}$$

As curve passes through the point (1, 1), so $1+1=c$

$\Rightarrow c=2$

$x^2+y^2-2x=0$, which is a circle of radius one.

**36.** **(b)** We have $(2+\sin x)\dfrac{dy}{dx}+(y+1)\cos x=0$

$$\Rightarrow\frac{d}{dx}(2+\sin x)(y+1)=0$$

On integrating, we get

$(2+\sin x)(y+1)=C$

At $x=0,\ y=1$ we have

$(2+\sin 0)(1+1)=C$

$\Rightarrow\ C=4$

$$\Rightarrow\ y+1=\frac{4}{2+\sin x}$$

$$y=\frac{4}{2+\sin x}-1$$

$$\text{Now } y\left(\frac{\pi}{2}\right)=\frac{4}{2+\sin\frac{\pi}{2}}-1=\frac{4}{3}-1=\frac{1}{3}$$

**37.** **(b)** $y(1+xy)dx=xdy$

$$\frac{xdy-ydx}{y^2}=xdx$$

$$\Rightarrow\int -d\left(\frac{x}{y}\right)=\int xdx$$

$$-\frac{x}{y}=\frac{x^2}{2}+C \text{ as } y(1)=-1\Rightarrow C=\frac{1}{2}$$

$$\text{Hence, } y=\frac{-2x}{x^2+1}\Rightarrow f\left(\frac{-1}{2}\right)=\frac{4}{5}$$

**38.** **(d)** Let $L=\lim_{t\to x}\dfrac{t^2f(x)-x^2f(t)}{t-x}=1$

Applying L.H. rule

$$L=\lim_{t\to x}\frac{2t\,f(x)-x^2f'(t)}{1}=1$$

$2x\,f(x)-x^2f'(x)=1$

solving above differential equation, we get

$$f(x)=\frac{2}{3}x^2+\frac{1}{3x}$$

Put $x=\dfrac{3}{2}$

$$f\left(\frac{3}{2}\right)=\frac{2}{3}\times\left(\frac{3}{2}\right)^2+\frac{1}{3}\times\frac{2}{3}=\frac{3}{2}+\frac{2}{9}=\frac{27+4}{18}=\frac{31}{18}$$

**39.** **(b)** Given differential equation is

$ydx-(x+2y^2)\,dy=0$

$\Rightarrow ydx-xdy-2y^2dy=0$

$$\Rightarrow\ \frac{ydx-xdy}{y^2}=2dy$$

$$\Rightarrow d\left(\frac{x}{y}\right)=2dy$$

Integrate both the side

$$\Rightarrow\frac{x}{y}=2y+c$$

using $f(-1)=1$, we get

$c = 1$

$\Rightarrow \frac{x}{y} = 2y + 1$

put $y = 1$, we get $f(a) = 3$

**40.** **(a)** $(x+2)\frac{dy}{dx} = x^2 + 4x - 9 \quad x \neq -2$

$$\frac{dy}{dx} = \frac{x^2+4x-9}{x+2}$$

$$dy = \frac{x^2+4x-9}{x+2}dx$$

$$\int dy = \int \frac{x^2+4x-9}{x+2}dx$$

$$y = \int\left(x+2-\frac{13}{x+2}\right)dx$$

$$y = \int(x+2)\,dx - 13\int\frac{1}{x+2}dx$$

$$y = \frac{x^2}{2} + 2x - 13\log|x+2| + c$$

Given that $y = (0) = 0$

$0 = -13\log 2 + c$

$$y = \frac{x^2}{2} + 2x - 13\log|x+2| + 13\log 2$$

$y(-4) = 8 - 8 - 13\log 2 + 13\log 2 = 0$

**41.** **(c)** Given differential equation is

$$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$$

By separating the variable, we get

$$dp(t) = \left[\frac{1}{2}p(t) - 200\right]dt$$

$$\Rightarrow \frac{dp(t)}{\frac{1}{2}p(t) - 200} = dt$$

Integrate on both the sides,

$$\int\frac{d(p(t))}{\frac{1}{2}p(t) - 200} = \int dt$$

Let $\frac{1}{2}p(t) - 200 = s \Rightarrow \frac{dp(t)}{2} = ds$

So, $\int\frac{d\,p(t)}{\left(\frac{1}{2}p(t) - 200\right)} = \int dt$

$$\Rightarrow \int\frac{2ds}{s} = \int dt \Rightarrow 2\log s = t + c$$

$$\Rightarrow 2\log\left(\frac{p(t)}{2} - 200\right) = t + c$$

$$\Rightarrow \frac{p(t)}{2} - 200 = e^{\frac{t}{2}}k$$

Using given condition $p(t) = 400 - 300\,e^{t/2}$

**42.** **(d)** Given $\frac{dy}{dx} = \frac{y}{x} + \phi\left(\frac{y}{x}\right)$ ...(1)

Let $\left(\frac{y}{x}\right) = v$ so that $y = xv$

or $\frac{dy}{dx} = x\frac{dv}{dx} + v$ ...(2)

from (1) & (2), $x\frac{dv}{dx} + \cancel{v} = \cancel{v} + \phi\left(\frac{1}{v}\right)$

$$\text{or, } \frac{dv}{\phi\left(\frac{1}{v}\right)} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int\frac{dx}{x} = \int\frac{dv}{\phi\left(\frac{1}{v}\right)} \Rightarrow \ln x + c = \int\frac{dv}{\phi\left(\frac{1}{v}\right)}$$

(where c being constant of integration)

But, given $y = \frac{x}{\ln|cx|}$ is the general solution

so that $\frac{x}{y} = \frac{1}{v} = \ln|cx| = \int\frac{dv}{\phi\left(\frac{1}{v}\right)}$

Differentiating w.r.t $v$ both sides, we get

$$\phi\left(\frac{1}{v}\right) = \frac{-1}{v^2} \Rightarrow \phi\left(\frac{x}{y}\right) = -\frac{y^2}{x^2}$$

when $\frac{x}{y} = 2$ i.e. $\phi(2) = -\left(\frac{y}{x}\right)^2 = -\left(\frac{1}{2}\right)^2 = \left(\frac{-1}{4}\right)$

**43.** **(c)** Given, Rate of change is $\frac{dP}{dx} = 100 - 12\sqrt{x}$

$\Rightarrow dP = (100 - 12\sqrt{x})\,dx$

By integrating

$$\int dP = \int(100 - 12\sqrt{x})\,dx$$

$P = 100x - 8x^{3/2} + C$

Given when $x = 0$ then $P = 2000$

$\Rightarrow C = 2000$

Now when $x = 25$ then

$P = 100 \times 25 - 8 \times (25)^{3/2} + 2000 = 4500 - 1000$

$\Rightarrow P = 3500$

**44.** **(a)** Slope $= \frac{dy}{dx} = 1 - \frac{1}{x^2}$

$\Rightarrow \int dy = \int \left(1 - \frac{1}{x^2}\right) dx$

$\Rightarrow y = x + \frac{1}{x} + C$, which is the equation of the curve

since curve passes through the point $\left(2, \frac{7}{2}\right)$

$\therefore \frac{7}{2} = 2 + \frac{1}{2} + C \Rightarrow C = 1$

$\therefore y = x + \frac{1}{x} + 1$

when $x = -2$, then $y = -2 + \frac{1}{-2} + 1 = \frac{-3}{2}$

**45.** **(d)** Given differential equation is

$\frac{dy}{dx} = \frac{y^3}{2(xy^2 - x^2)}$

By substituting $z = y^2$, we get diff. eqn. as

$\frac{dz}{dx} = \frac{2z^2}{2(xz - x^2)} = \frac{z^2}{xz - x^2}$

Now, $\frac{dx}{dz} = \frac{x}{z} - \frac{x^2}{z^2} = \frac{x}{z}\left[1 - \frac{x}{z}\right] \approx F\left(\frac{x}{z}\right)$

Hence, statement-1 is true.

Now, $y^2 e^{-y^2/x} = C$ satisfies the given diff. equation

$\therefore$ It is the solution of given diff. equation.

Thus, statement-2 is also true.

**46.** **(a)** Given differential equation is

$\frac{dp(t)}{dt} = 0.5\,p(t) - 450$

$\Rightarrow \frac{dp(t)}{dt} = \frac{1}{2}p(t) - 450$

$\Rightarrow \frac{dp(t)}{dt} = \frac{p(t) - 900}{2}$

$\Rightarrow 2\frac{dp(t)}{dt} = -[900 - p(t)]$

$\Rightarrow 2\frac{dp(t)}{900 - p(t)} = -dt$

Integrate both the side, we get

$-2\int \frac{dp(t)}{900 - p(t)} = \int dt$

Let $900 - p(t) = u$

$\Rightarrow -dp(t) = du$

$2\int \frac{du}{u} = \int dt \Rightarrow 2\ln u = t + c$ ...(i)

$\Rightarrow 2\ln[900 - p(t)] = t + c$

Given $t = 0, p(0) = 850$

$2\ln(50) = c$

Putting in (i)

$\Rightarrow 2\left[\ln\left(\frac{900 - p(t)}{50}\right)\right] = t$

$\Rightarrow 900 - p(t) = 50e^{\frac{t}{2}}$

$\Rightarrow p(t) = 900 - 50e^{\frac{t}{2}}$

let $p(t_1) = 0$

$0 = 900 - 50e^{\frac{t_1}{2}} \qquad \therefore t_1 = 2\ln 18$

**47.** **(c)** Given differential equation is

$\frac{(2 + \sin x)}{(1 + y)} \cdot \frac{dy}{dx} = \cos x$

which can be rewritten as

$\frac{dy}{1 + y} = \frac{\cos x}{2 + \sin x} dx$

Integrate both the sides, we get

$\int \frac{dy}{1 + y} = \int \frac{\cos x\, dx}{2 + \sin x}$

$\Rightarrow \log(1 + y) = \log(2 + \sin x) + \log C$

$\Rightarrow 1 + y = C(2 + \sin x)$

Given $y(0) = 2$

$\Rightarrow 1 + 2 = C[2 + \sin 0] \Rightarrow C = \frac{3}{2}$

Now, $y\left(\frac{\pi}{2}\right)$ can be found as

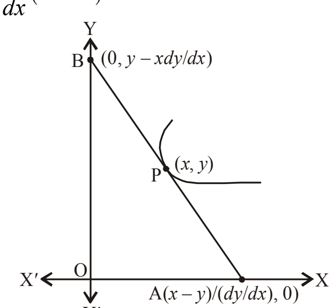
$1 + y = \frac{3}{2}\left(2 + \sin\frac{\pi}{2}\right) \Rightarrow 1 + y = \frac{9}{2}$

$\Rightarrow y = \frac{7}{2}$

Hence, $y\left(\frac{\pi}{2}\right) = \frac{7}{2}$

**48.** **(b)** Equation of tangent at P

$Y - y = \frac{dy}{dx}(X - x)$




$X$-intercept $= x - \frac{y}{dy/dx}$

$Y$-intercept $= y - \frac{xdy}{dx}$

Since P is mid-point of A and B

$x-\frac{y}{dy}=2x$ and $y-\frac{xdy}{dx}=2y$

$\Rightarrow \frac{-y}{\frac{dy}{dx}}=x$ and $\frac{-xdy}{dx}=y$

$\Rightarrow \frac{dx}{x}=-\frac{dy}{y}$

$\ell ny=-\ell nx+\ell nc$

$y=\frac{c}{x}$

Since the above line passes through the point (2, 3).

$\therefore \quad c=6$

Hence $y=\frac{6}{x}$ is the required equation.

**49.** **(a)** $\frac{dV(t)}{dt}=-k(T-t)$

$\Rightarrow \int dV(t)=-k\int(T-t)dt$

$V(t)=\frac{k(T-t)^2}{2}+c$

at $t=0$, $V(t)=I$

$I=\frac{kT^2}{2}+c$

$\Rightarrow c=I-\frac{kT^2}{2}$

$\Rightarrow V(t)=I+\frac{k}{2}(t^2-2tT)$

$V(T)=I+\frac{k}{2}(T^2-2T^2)\ =I-\frac{k}{2}T^2$

**50.** **(d)** $\frac{dy}{dx}=y+3 \Rightarrow \int\frac{dy}{y+3}=\int dx$

$\Rightarrow \ell n|y+3|=x+c$

Given $y(0)=2$, $\therefore \ell n5=c$

$\Rightarrow \ell n|y+3|=x+\ell n5$

Put $x=\ell n 2$, then $\ell n|y+3|=\ell n2+\ell n5$

$\Rightarrow \ell n|y+3|=\ell n10$

$\therefore\ y+3=\pm10 \Rightarrow y=7,-13$

**51.** **(d)** $\frac{dy}{dx}=\frac{x+y}{x}=1+\frac{y}{x}$

It is homogeneous differential eqn.

Putting $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

we get

$v+x\frac{dv}{dx}=1+v \Rightarrow \int\frac{dx}{x}=\int dv$

$\Rightarrow v=\ln x+c \Rightarrow y=x\ln x+cx$

As $y(1)=1$

$\therefore\ c=1$ So solution is $y=x\ln x+x$

**52.** **(b)** Equation of normal at $P(x,y)$ is

$Y-y=-\frac{dx}{dy}(X-x)$

Coordinate of $G$ at $X$ axis is $(X,0)$ (let)

$\therefore\ 0-y=-\frac{dx}{dy}(X-x)$

$\Rightarrow\ y\frac{dy}{dx}=X-x$

$\Rightarrow\ X=x+y\frac{dy}{dx}$

$\therefore$ Co-ordinate of $G\left(x+y\frac{dy}{dx},0\right)$

Given distance of $G$ from origin = twice of the abscissa of $P$.

$\because$ distance cannot be –ve, therefore abscissa $x$ should be +ve

$\therefore\ x+y\frac{dy}{dx}=2x \Rightarrow y\frac{dy}{dx}=x$

$\Rightarrow\ ydy=xdx$

On Integrating, we have $\frac{y^2}{2}=\frac{x^2}{2}+c_1$

$\Rightarrow\ x^2-y^2=-2c_1$

$\therefore$ the curve is a hyperbola

**53.** **(c)** $\frac{xdy}{dx}=y(\log y-\log x+1)$

$\frac{dy}{dx}=\frac{y}{x}\left(\log\left(\frac{y}{x}\right)+1\right)$

Put $y=vx$

$\frac{dy}{dx}=v+\frac{xdv}{dx} \Rightarrow v+\frac{xdv}{dx}=v(\log v+1)$

$\frac{xdv}{dx}=v\log v \Rightarrow \int\frac{dv}{v\log v}=\int\frac{dx}{x}$

Put $\log v=z$

$\frac{1}{v}dv=dz \Rightarrow \int\frac{dz}{z}=\int\frac{dx}{x}$

$\ln z=\ln x+\ln c$

$x=cx$ or $\log v=cx$ or $\log\left(\frac{y}{x}\right)=cx.$

**54.** **(b)** $\frac{d^2y}{dx^2}=e^{-2x}$; on integration $\frac{dy}{dx}=\frac{e^{-2x}}{-2}+c$;

Again integrate we get $y=\frac{e^{-2x}}{4}+cx+d$

**55.** **(c)** $\frac{dy}{dx}+2y\tan x=2\sin x$

$$\text{I.F.} = e^{\int 2\tan x\,dx} = \sec^2 x$$

The solution of the differential equation is

$$y \times \text{I.F.} = \int \text{I.F.} \times 2\sin x\,dx + C$$

$$\Rightarrow y \cdot \sec^2 x = \int 2\sin x \cdot \sec^2 x\,dx + C$$

$$\Rightarrow y\sec^2 x = 2\sec x + C \quad ...(i)$$

When $x = \frac{\pi}{3}$, $y = 0$; then $C = -4$

$\therefore$ From (i), $y\sec^2 x = 2\sec x - 4$

$$\Rightarrow y = \frac{2\sec x - 4}{\sec^2 x} \Rightarrow y\left(\frac{\pi}{4}\right) = \sqrt{2} - 2$$

**56. (a)** $\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x)$

$$\text{I.F.} = e^{-\int \frac{1}{x}dx} = \frac{1}{x}$$

$$\therefore \int d\left(\frac{y}{x}\right) = \int (x\cos x + \sin x)dx$$

$$\Rightarrow \frac{y}{x} = x\sin x + C \qquad \because y(\pi) = \pi \Rightarrow C = 1$$

$$y = x^2\sin x + x \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$$

$$y' = 2x\sin x + x^2\cos x + 1$$

$$y'' = 2\sin x - x^2\sin x \Rightarrow y''\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4}$$

$$\therefore y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4} + \frac{\pi^2}{4} + \frac{\pi}{2} = 2 + \frac{\pi}{2}$$

**57. (c)** $(x+1)dy = ((x+1)^2 + (y-3))dx = 0$

$$\Rightarrow \frac{dy}{dx} = (1+x) + \left(\frac{y-3}{1+x}\right)$$

$$\frac{dy}{dx} - \frac{1}{(1+x)}y = (1+x) - \frac{3}{(1+x)}$$

$$\text{I.F.} = e^{-\int \frac{1}{(1+x)}dx} = \frac{1}{(1+x)}$$

$$\therefore \frac{d}{dx}\left(\frac{y}{1+x}\right) = 1 - \frac{3}{(1+x)^2}$$

$$y = (1+x)\left[x + \frac{3}{(1+x)} + C\right]$$

$\because$ At $x = 2, y = 0$

$\therefore$ $0 = 3(2 + 1 + C) \Rightarrow C = -3$

Then, $y = (1+x)\left[x + \frac{3}{1+x} - 3\right]$

Now, at $x = 3$, $y = (1+3)\left[3 + \frac{3}{1+3} - 3\right] = 3$

**58. (a)** The given differential equation is $\frac{dx}{dy} + x = y^2$

Comparing with $\frac{dx}{dy} + Px = Q$, where $P = 1, Q = y^2$

Now, $\text{I.F.} = e^{\int 1.dy} = e^y$

$$x.e^y = \int (y^2)e^y.dy = y^2.e^y - \int 2y.e^y.dy$$

$$= y^2e^y - 2(y.e^y - e^y) + C$$

$$\Rightarrow x.e^y = y^2e^y - 2ye^y + 2e^y + C$$

$$\Rightarrow x = y^2 - 2y + 2 + C.e^{-y} \quad ...(i)$$

As $y(0) = 1$, satisfying the given differential eqn,

$\therefore$ put $x = 0, y = 1$ in eqn. (i)

$$0 = 1 - 2 + 2 + \frac{C}{e}$$

$C = -e$

$y = 0, x = 0 - 0 + 2 + (-e)(e^{-0})$

$x = 2 - e$

**59. (b)** Consider the differential equation,

$$y^2dx + \left(x - \frac{1}{y}\right)dy = 0$$

$$\Rightarrow \frac{dx}{dy} + \left(\frac{1}{y^2}\right)x = \frac{1}{y^3}$$

$$\text{I.F.} = e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$$

$$\therefore x.e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}} \frac{1}{y^3}dy + c$$

Put $-\frac{1}{y} = u \Rightarrow \frac{1}{y^2}dy = du$

$$\Rightarrow x.e^{-\frac{1}{y}} = -\int ue^u du + c = -ue^u + e^u + c$$

$$\Rightarrow x.e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(\frac{1}{y} + 1\right) + c$$

At $y = 1, x = 1$

$$1 = 2 + ce \Rightarrow c = -\frac{1}{e} \Rightarrow x = \left(1+\frac{1}{y}\right) - \frac{1}{e}e^{\frac{1}{y}}$$

On putting $y = 2$, we get $x = \frac{3}{2} - \frac{1}{\sqrt{e}}$

**60.** **(a)** $\frac{dy}{dx} + y\sec^2 x = \sec^2 x \times \tan x$

Given equation is linear differential equation.

$$\text{IF} = e^{\int \sec^2 x dx} = e^{\tan x}$$

$$\Rightarrow y.e^{\tan x} = \int e^{\tan x}\sec^2 x \times \tan x dx$$

Put $\tan x = u = \sec^2 x \, dx = du$

$$ye^{\tan x} = \int e^u u \, du \Rightarrow ye^{\tan x} = ue^u - e^u + c$$

$$\Rightarrow ye^{\tan x} = (\tan x - 1)\, e^{\tan x} + c$$

$$\Rightarrow y = (\tan x - 1) + c.\, e^{-\tan x}$$

$\therefore\ y(0) = 0$ (given) $\Rightarrow 0 = -1 + c \Rightarrow c = 1$

Hence, solution of differential equation,

$$y\left(-\frac{\pi}{4}\right) = -1 - 1 + e = -2 + e$$

**61.** **(d)** Given differential equation is,

$$\frac{dy}{dx} + y\tan x = 2x + x^2\tan x$$

Here, $P = \tan x$, $Q = 2x + x^2\tan x$

$$\text{I.F.} = e^{\int \tan x dx} = e^{\ln|\sec x|} = |\sec x|$$

$$\therefore\ y(\sec x) = \int (2x + x^2\tan x)\sec x dx$$

$$= \int x^2\tan x\sec x dx + \int 2x\sec x dx = x^2\sec x + c$$

Given $y(0) = 1 \Rightarrow c = 1$

$\therefore\ y = x^2 + \cos x$ ...(i)

Now put $x = \frac{\pi}{4}$ and $x = \frac{-\pi}{4}$ in equation (i),

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}} \text{ and } y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$$

$$\Rightarrow y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = 0$$

$$\frac{dy}{dx} = 2x - \sin x$$

$$\therefore y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}} \text{ and } y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$$

$$\Rightarrow y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$$

**62.** **(c)** $\frac{dy}{dx} + \frac{2}{x}y = x$ $y(1) = 1$ (given)

Since, the above differential equation is the linear differential equation, then $I.F = e^{\int \frac{2}{x}dx} = x^2$

Now, the solution of the linear differential equation

$$y \times x^2 = \int x^3 dx$$

$$\Rightarrow yx^2 = \frac{x^4}{4} + C$$

$\because y(1) = 1$

$$\therefore 1 \times 1 = \frac{1}{4} + C \Rightarrow C = \frac{3}{4}$$

$\therefore$ Solution becomes.

$$y = \frac{x^2}{4} + \frac{3}{4x^2}$$

**63.** **(d)** $(1+x^2)^2 \frac{dy}{dx} + 2x(1+x^2)y = 1$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{1}{(1+x^2)^2}$$

Since, the above differential equation is a linear differential equation

$$\therefore \quad \text{I.F.} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1 + x^2$$

Then, the solution of the differential equation

$$\Rightarrow y(1+x^2) = \int \frac{dx}{1+x^2} + c$$

$$\Rightarrow y(1+x^2) = \tan^{-1} x + c \quad ...(1)$$

If $x = 0$ then $y = 0$ (given)

$\Rightarrow$ $0 = 0 + c$

$\Rightarrow$ $c = 0$

Then, equation (1) becomes,

$\Rightarrow$ $y(1+x^2) = \tan^{-1} x$

Now put $x = 1$ in above equation, then

$$2y = \frac{\pi}{4}$$

$$\Rightarrow 2\left(\frac{\pi}{32\sqrt{a}}\right) = \frac{\pi}{4} \qquad \left[\sqrt{a}\,y(1) = \frac{\pi}{32}\right]$$

$\Rightarrow \quad \sqrt{a} = \dfrac{1}{4}$

$\Rightarrow \quad a = \dfrac{1}{16}$

**64.** **(c)** Consider the differential equation,

$$\frac{dy}{dx} + \frac{y}{x} = \log_e x$$

$\because \quad IF = e^{\int \frac{1}{x} dx} = x$

$\therefore \quad yx = \int x \ln x \, dx$

$\Rightarrow \quad xy = \ln x \cdot \dfrac{x^2}{2} - \displaystyle\int \frac{1}{x} \cdot \frac{x^2}{2} dx$

$\Rightarrow \quad xy = \dfrac{x^2}{2} \cdot \ln x - \dfrac{x^2}{4} + c$

Given, $2y(2) = \log_e 4 - 1$.

$\therefore \quad 2y = 2\ln 2 - 1 + c$

$\Rightarrow \quad \ln 4 - 1 = \ln 4 - 1 + c$

i.e. $\quad c = 0$

$\Rightarrow \quad xy = \dfrac{x^2}{2} \ln x - \dfrac{x^2}{4}$

$\Rightarrow \quad y = \dfrac{x}{2} \ln x - \dfrac{x}{4}$

$\Rightarrow \quad y(e) = \dfrac{e}{2} - \dfrac{e}{4} = \dfrac{e}{4}$

**65.** **(b)** $\because$ Slope of the tangent $= \dfrac{x^2 - 2y}{x}$

$\because \quad \dfrac{dy}{dx} = \dfrac{x^2 - 2y}{x}$

$\dfrac{dy}{dx} + \dfrac{2}{x} y = x$

I.F. $= e^{\int \frac{2}{x} dx} = e^{2\ln x} = x^2$

Solution of equation

$y \cdot x^2 = \int x \cdot x^2 dx$

$x^2 y = \dfrac{x^4}{4} + C$

$\therefore$ curve passes through point (1, –2)

$(1)^2(-2) = \dfrac{1^4}{4} + C$

$\Rightarrow \quad C = \dfrac{-9}{4}$

Then, equation of curve

$y = \dfrac{x^2}{4} - \dfrac{9}{4x^2}$

Since, above curve satisfies the point.

Hence, the curve passes through $(\sqrt{3}, 0)$.

**66.** **(c)** Given differential equation is,

$$\frac{dy}{dx} + \left(2 + \frac{1}{x}\right) y = e^{-2x}, x > 0$$

$\text{IF} = e^{\int \left(2 + \frac{1}{x}\right) dx} = e^{2x + \ln x} = xe^{2x}$

Complete solution is given by

$y(x) \cdot x\, e^{2x} = \int x\, e^{2x} \cdot e^{-2x} dx + c$

$= \int x \, dx + c$

$y(x) \cdot e^{2x} \cdot x = \dfrac{x^2}{2} + c$

Given, $y(1) = \dfrac{1}{2} e^{-2}$

$\therefore \quad \dfrac{1}{2} e^{-2} \cdot e^2 \cdot 1 = \dfrac{1}{2} + c \Rightarrow c = 0$

$\therefore \quad y(x) = \dfrac{x^2}{2} \cdot \dfrac{e^{-2x}}{x}$

$y(x) = \dfrac{x}{2} \cdot e^{-2x}$

Differentiate both sides with respect to $x$,

$y'(x) = \dfrac{e^{-2x}}{2}(1 - 2x) < 0 \;\forall\; x \in \left(\dfrac{1}{2}, 1\right)$

Hence, $y(x)$ is decreasing in $\left(\dfrac{1}{2}, 1\right)$

**67.** **(b)** Let $y = f(x)$

$\dfrac{dy}{dx} + \left(\dfrac{3}{4x}\right) y = 7$

I.F. $= e^{\int \frac{3}{4x} dx} = e^{\frac{3}{4} \ln x} = x^{\left(\frac{3}{4}\right)}$

Solution of differential equation

$y \cdot x^{\frac{3}{4}} = \int 7 \cdot x^{\frac{3}{4}} dx + C$

$y \cdot x^{\frac{3}{4}} = 7 \cdot \dfrac{x^{\frac{7}{4}}}{\left(\frac{7}{4}\right)} + C = 4x^{\frac{7}{4}} + C$

$y = 4x + Cx^{-\frac{3}{4}}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{4}{x} + Cx^{\frac{3}{4}}$

$\Rightarrow \lim_{x\to 0^+} x \cdot f\left(\frac{1}{x}\right) = \lim_{x\to 0^+}\left(4 + Cx^{\frac{7}{4}}\right) = 4$

**68.** **(c)** Since, $x\frac{dy}{dx} + 2y = x^2$

$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = x$

I.F. $= e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$.

Solution of differential equation is:

$y \cdot x^2 = \int x \cdot x^2 dx$

$y \cdot x^2 = \frac{x^4}{4} + C$ ...(1)

$\because$ $y(1) = 1$

$\therefore$ $C = \frac{3}{4}$

Then, from equation (1)

$y \cdot x^2 = \frac{x^4}{4} + \frac{3}{4}$

$\therefore$ $y = \frac{x^2}{4} + \frac{3}{4x^2}$

$\therefore$ $y\left(\frac{1}{2}\right) = \frac{1}{16} + 3 = \frac{49}{16}$

**69.** **(b)** Consider the given differential equation the sinxdy + ycosxdx = 4xdx

$\Rightarrow$ d(y.sinx) = 4xdx

Integrate both sides

$\Rightarrow$ y.sinx $= 2x^2 + C$ ...(1)

$\Rightarrow$ $y(x) = \frac{2x^2}{\sin x} + c$ ...(2)

$\because$ eq. (2) passes through $\left(\frac{\pi}{2}, 0\right)$

$\Rightarrow 0 = \frac{\pi^2}{2} + C$

$\Rightarrow C = -\frac{\pi^2}{2}$

Now, put the value of C in (1)

Then, $y\sin x = 2x^2 - \frac{\pi^2}{2}$ is the solution

$\therefore$ $y\left(\frac{\pi}{6}\right) = \left(2.\frac{\pi^2}{36} - \frac{\pi^2}{2}\right)2 = -\frac{8\pi^2}{9}$

**70.** **(a)** When $x \in [0, 1]$, then $\frac{dy}{dx} + 2y = 1 \Rightarrow y = \frac{1}{2} + C_1 e^{-2x}$

$\because y(0) = 0 \Rightarrow y(x) = \frac{1}{2} - \frac{1}{2}e^{-2x}$

Here, $y(1) = \frac{1}{2} - \frac{1}{2}\, e^{-2} = \frac{e^2 - 1}{2e^2}$

When $x \notin [0, 1]$, then $\frac{dy}{dx} + 2y = 0 \Rightarrow y = c_2\, e^{-2x}$

$\because y(1) = \frac{e^2 - 1}{2} \Rightarrow \frac{e^2 - 1}{2} = c^2 e^{-2} \Rightarrow C_2 = \frac{e^2 - 1}{2}$

$\therefore y(x)\left(\frac{e^2 - 1}{2}\right)e^{-2x} \Rightarrow y\left(\frac{3}{2}\right) = \frac{e^2 - 1}{2e^3}$

**71.** **(b)** $y\,dx - x\,dy - 3y^2\,dy = 0$

$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 3y \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 3y$

if $= e^{-\int \frac{1}{y}dy} = e^{-\ln y} = \frac{1}{y}$

$\therefore$ solution is $\frac{x}{y} = \int 3y \cdot \frac{1}{y}dy$

$\Rightarrow \frac{x}{y} = 3y + c$

which passes through (1, 1)

$\therefore$ $1 = 3 + c \Rightarrow c = -2$

$\therefore$ solution becomes

$\Rightarrow$ $x = 3y^2 - 2y$

which also passes through $\left(-\frac{1}{3}, \frac{1}{3}\right)$

**72.** **(d)** $\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$

$2y\frac{dy}{dx} + y^2 \sec x = \tan x$

Put $y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + t\sec x = \tan x$

$I.f = e^{\int \sec x dx} = e^{\ln(\sec x + \tan x)} = \sec x + \tan x$

$$\frac{dt}{dx}(\sec x+\tan x)+t\sec x(\sec x+\tan x)$$
$$=\tan x(\sec x+\tan x)$$
$$\int d(t(\sec x+\tan x))=\int \tan x(\sec x+\tan x)\,dx$$
$$t(\sec x+\tan x)=\sec x+\tan x-x$$
$$t=1-\frac{x}{\sec x+\tan x}\Rightarrow y^2=1-\frac{x}{\sec x+\tan x}$$

**73. (a)** Given, $\frac{dy}{dx}+\left(\frac{1}{x\log x}\right)y=2$

$\text{I.F.}=e^{\int\frac{1}{x\log x}dx}=e^{\log(\log x)}=\log x$

$y.\log x=\int 2\log x\,dx+c$

$y\log x=2[x\log x-x]+c$

Put $x=1$, $y.0=-2+c$

$c=2$

Put $x=e$

$y\log e=2e(\log e-1)+c$

$y(e)=c=2$

**74. (c)** Let $\frac{dy}{dx}+y\tan x=\sin 2x$

$\text{I.F}=e^{\int\tan x\,dx}=e^{-\log\cos x}=\sec x$

Required solution is

$y(\sec x)=\int\sin 2x\sec x\,dx+c$

$y(\sec x)=\int\frac{2\sin x\cos x}{\cos x}dx+c$

$y(\sec x)=2\int\sin x\,dx+c$

$y(\sec x)=-2\cos x+c$ ...(1)

Given $y(0)=1$

$\therefore$ put $x=0$ and $y=1$, we get

$1(\sec 0)=-2\cos 0+c$

$\Rightarrow c=1+2\Rightarrow c=3$

$\therefore$ from eqn (1), we have

$y\sec x=-2\cos x+3$ ...(2)

To find $y(\pi)$, put $x=\pi$ in eqn (2), we get

$y(\sec\pi)=-2\cos\pi+3$

$y=-2(-1)(-1)+3(-1)=-2-3=-5$

**75. (d)** Given, $\sin 2x\left(\frac{dy}{dx}-\sqrt{\tan x}\right)-y=0$

or, $\frac{dy}{dx}=\frac{y}{\sin 2x}+\sqrt{\tan x}$

or, $\frac{dy}{dx}-y\,\text{cosec}\,2x=\sqrt{\tan x}$ ...(1)

Now, integrating factor (I.F) $=e^{\int-\text{cosec}2x}$

or, $\text{I.F}=e^{-\frac{1}{2}\log|\tan x|}=e^{\log\left(\sqrt{\tan x}\right)^{-1}}$

$=\frac{1}{\sqrt{\tan x}}=\sqrt{\cot x}$

Now, general solution of eq. (1) is written as

$y(\text{I.F.})=\int Q(\text{I.F.})\,dx+c$

$\therefore\ y\sqrt{\cot x}=\int\sqrt{\tan x}.\sqrt{\cot x}\,dx+c$

$\therefore\ y\sqrt{\cot x}=\int 1.dx+c$

$\boxed{\therefore\ y\sqrt{\cot x}=x+c}$

**76. (d)** Given differential equation is

$(1+x^2)\frac{dy}{dx}+2xy=4x^2$

$\Rightarrow\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{4x^2}{1+x^2}$

This is linear diff. equation

$\text{I.F}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$

$\therefore$ Solution is

$y(1+x^2)=\int\frac{4x^2}{1+x^2}\times 1+x^2+C$

$\Rightarrow y(1+x^2)=\frac{4x^3}{3}+C$

$\Rightarrow$ Required curve is

$3y(1+x^2)=4x^3\ (\because\ C=0)$

**77. (b)** Given differential equation is $(x^2-1)\frac{dy}{dx}+2xy=x$

$\Rightarrow\frac{dy}{dx}+\frac{2x}{x^2-1}.y=\frac{x}{x^2-1}$

This is in linear form.

Integrating factor $=e^{\int\frac{2x}{x^2-1}dx}=e^{\int\frac{dt}{t}}$ where $t=x^2-1$

$=e^{\log t}=x^2-1$

Hence, required integrating factor $=x^2-1$.

**78. (d)** Given differential equation is

$\frac{dy}{dx}+\frac{2}{x}.y=x^2$

This is of the linear form.

$\therefore\ P=\frac{2}{x},\ Q=x^2$

$\text{I.F}=e^{\int\frac{2}{x}dx}=e^{\log x^2}=x^2$

Solution is

$$y.x^2 = \int x^2.x^2 dx + c = \frac{x^5}{5} + c$$

$$y = \frac{x^3}{5} + cx^{-2}$$

**79.** **(c)** $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

It is linear differential eqn.

$$\text{I.F.} = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$$

$$\text{So} \quad x.e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$$

$$\text{Let} \quad \frac{-1}{y} = t$$

$$\Rightarrow \quad \frac{1}{y^2} dy = dt$$

$$\Rightarrow \quad I = -\int te^t dt = e^t - te^t \quad = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$$

$$\Rightarrow \quad xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$$

$$\Rightarrow \quad x = 1 + \frac{1}{y} + c.e^{1/y}$$

Given $y(1) = 1$

$$\therefore \quad c = -\frac{1}{e}$$

$$\Rightarrow \quad x = 1 + \frac{1}{y} - \frac{1}{e}.e^{1/y}$$

**80.** **(d)** $\cos x\, dy = y(\sin x - y)\, dx$

$$\frac{dy}{dx} = y \tan x - y^2 \sec x$$

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x \quad \text{...(i)}$$

$$\text{Let } \frac{1}{y} = t \Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

Putting in (i)

$$-\frac{dt}{dx} - t \tan x = -\sec x$$

$$\Rightarrow \frac{dt}{dx} + (\tan x)t = \sec x$$

$$\text{I.F.} = e^{\int \tan x\, dx} = e^{\log|\sec x|} = \sec x$$

$$\text{Solution : } t \sec x = \int \sec x \sec x\, dx$$

$$\Rightarrow \frac{1}{y}\sec x = \tan x + c$$

**81.** **(b)** $ydx + (x + x^2 y)dy = 0$

$$\Rightarrow \frac{dx}{dy} = -\frac{x}{y} - x^2 \Rightarrow \frac{dx}{dy} + \frac{x}{y} = -x^2,$$

It is Bernoulli form. Divide by $x^2$

$$x^{-2}\frac{dx}{dy} + x^{-1}\left(\frac{1}{y}\right) = -1.$$

$$\text{put } x^{-1} = t, -x^{-2}\frac{dx}{dy} = \frac{dt}{dy} \text{ we get,}$$

$$-\frac{dt}{dy} + t\left(\frac{1}{y}\right) = -1 \Rightarrow \frac{dt}{dy} - \left(\frac{1}{y}\right)t = 1$$

It is linear differential eqn. in $t$.

$$\text{I.F} = e^{\int -\frac{1}{y} dy} = e^{-\log y} = y^{-1}$$

$$\therefore \text{Solution is } t(y^{-1}) = \int (y^{-1})dy + C$$

$$\Rightarrow \frac{1}{x}.\frac{1}{y} = \log y + C \Rightarrow \log y - \frac{1}{xy} = C$$

**82.** **(c)** $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{\tan^{-1} y}}{(1+y^2)}$$

It is form of linear differential equation.

$$I.F = e^{\int \frac{1}{(1+y^2)} dy} = e^{\tan^{-1} y}$$

$$x(e^{\tan^{-1} y}) = \int \frac{e^{\tan^{-1} y}}{1+y^2} e^{\tan^{-1} y} dy$$

$$x(e^{\tan^{-1} y}) = \frac{e^{2\tan^{-1} y}}{2} + C \qquad \left[\because \int e^{2x} dx = \frac{e^{2x}}{2}\right]$$

$$\therefore \; 2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$$